8.2 Arithmetic and Geometric Series

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Arithmetic

Sequence: $4, 7, 10, 13, \ldots$ (+3, +3, +3) $a_n = a_1 + (n-1)d$

a_1, a_2, a_3, a_4

Series: $4 + 7 + 10 + 13 = 34$

$a_1\ a_2\ a_3\ a_4$ $\quad S_4 = 34$

$$S_n = n \cdot \left(\frac{a_1 + a_n}{2}\right)$$

n = # of terms; a_1 = 1st term; a_n = last term; S_n = the sum of the first n terms

Find the sum of the first 100 terms.

$$S_{100} = 100 \cdot \left(\frac{4 + 301}{2}\right) = 15{,}250$$

$$a_{100} = 4 + (100-1)\cdot 3 = 301$$

Geometric

Sequence: $2, 10, 50, 250, \ldots$ (×5, ×5, ×5) $a_n = a_1 \cdot r^{(n-1)}$

$a_1\ a_2\ a_3\ a_4$

Series: $2 + 10 + 50 + 250 = 312$

$a_1\ a_2\ a_3\ a_4$

$$S_n = a_1 \left(\frac{1 - r^n}{1 - r}\right)$$

a_1 = 1st term; n = # of terms; S_n = the sum of the first n terms; r = ratio between terms

Find the sum of the first 8 terms.

$2 + 10 + 50 + 250 + 1250 + 6250 + 31250 + 156{,}250$

(a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8)

195,312

$$S_8 = 2 \cdot \left(\frac{1 - 5^8}{1 - 5}\right) = 2\left(\frac{-390624}{-4}\right) = 195{,}312$$

Summation Notation:

$$\sum_{x=1}^{n} f(x)$$

last value of x; $f(x)$ ← function in terms of x; $x=1$ ← first value of x

$$\sum_{x=1}^{6} (2x-1) = \underbrace{(2\cdot1-1)\ (2\cdot2-1)\ (2\cdot3-1)\ (2\cdot4-1)\ (2\cdot5-1)\ (2\cdot6-1)}_{\text{plug in } x=1 \to x=6 \text{ into } (2x-1)}$$

$x=1$ $x=2$ $x=3$ $x=4$ $x=5$ $x=6$

$1 + 3 + 5 + 7 + 9 + 11 = 36$

$$\sum_{p=1}^{4} (p^2 - 2p) = (1^2-2\cdot1)\ (2^2-2\cdot2)\ (3^2-2\cdot3)\ (4^2-2\cdot4)$$

$p=1$ $p=2$ $p=3$ $p=4$

$-1 + 0 + 3 + 8 = 10$

ex $\displaystyle\sum_{x=1}^{300} x = \frac{300(300+1)}{2} = 45{,}150$

$$\sum_{x=1}^{n} x = \frac{n(n+1)}{2}$$

$$\sum_{x=1}^{120} x^2 = \frac{120(120+1)(2\cdot120+1)}{6} = 583{,}220$$

$$\sum_{x=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}$$